Test: Introduction to Inference: Confidence Intervals and Hypothesis Testing

Instructor's Version

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Free Response: 2 questions, 52 points, 30 minutes

1. An advertisement for a word-processing class claims that students who complete the class will, on average, be able to type 60 words per minute (wpm) with a standard deviation of 6 wpm. At the end of the class, 49 students are tested and their mean score is 58.5 wpm.

A. Is this evidence that the true mean is *different* from 60 wpm? Give a complete answer, using a significance level of .05. (12 points)

Answer:

$H_0 : \mu = 60$ (The true mean is 60 wpm.)

$H_a : \mu \neq 60$ (The true mean is different from 60 wpm.)

Samples of size 49 are quite large, so we're justified in assuming that the sampling distribution of the sample mean is normal.

$$z_{58.5} = \frac{58.5 - 60}{6/\sqrt{49}} = -1.75 \rightarrow p = 2(.04) = .08.$$

(Note to instructors: on the TI-83 it's 2(normalcdf(−10,−1.75)). A *P*-value of .08, though not large, is larger than .05 and isn't sufficient evidence to reject our null hypothesis. Using this analysis, we don't have strong evidence to conclude that the performance of graduates differs significantly from 60 wpm.)

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B. Suppose the person conducting the research had believed, before collecting data, that the graduates weren't as good as claimed. Would this belief have changed the analysis you performed in part A? (12 points)

Answer:
It would change it. Now you have a one-sided alternative: $H_a : \mu < 60$. The computation of $z = -1.75$ is unchanged but the *P*-value, since the hypothesis is now one-sided, is just .04. This could change our conclusion. Now there's evidence, since we have a small *P*-value, that this difference is too large to attribute to chance and we have evidence that the graduates do type worse than 60 wpm.

C. Construct a 99% confidence interval for the true mean of the population. What conclusion can you draw from this interval, and do you have evidence to reject the claim that the average graduate can type 60 wpm? (6 points)

Answer:

$$58.5 \pm 2.58\left(\frac{6}{\sqrt{49}}\right) = (56.29, 60.71)$$

We note that 60 is in this interval. That is, 58.5 is a likely sample outcome from a population whose mean is 60. So using this analysis, we do not have evidence to reject the claim that the average graduate can type 60 wpm.

2. Consider the one-sided situation in problem 1: An advertisement for a word-processing class claims that students who complete the class will, on average, be able to type 60 words per minute (wpm) with a standard deviation of 6 wpm. We're concerned that students aren't performing as well as advertised. At the end of the class we test 49 students and conduct a one-sided hypothesis test at the .05 level of significance. This means that the hypothesis will be rejected if the mean wpm is less than 58.6. Suppose, in reality, the true mean for graduates is 58 wpm with a standard deviation of 5 wpm.

A. What's the probability of a Type II error? (14 points)

Answer:
We're given the decision rule, which is to reject the null hypothesis whenever the number of words per minute for the group of 49 students is less than 58.6 ($\bar{x} < 58.6$). We need to determine what this means in terms of the *real* sampling distribution, which we take as *N*(58,5/7). For this distribution, *P*(Type II error) = *P*(get a sample mean of greater than 58.6) = .20.

(Note to instructors: On a TI-83 this would be normalcdf(58.6,100,58,5/7) = .20.)

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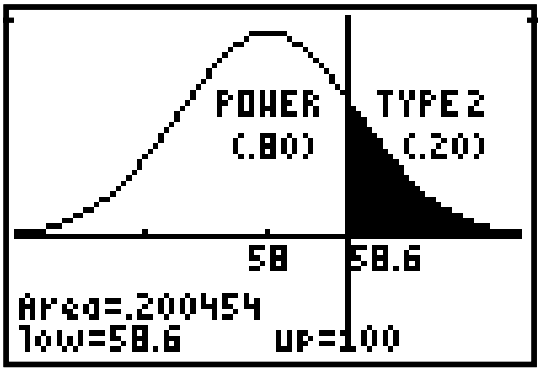
B. What's the power of the test if the true population mean is 58 wpm? Draw a diagram of the sampling distribution for the sample mean, showing the area that represents the power of the test and the area that represents the probability of a Type II error. (8 points)

Answer:
Power = 1 – *P*(Type II) = .80



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